

Code No: 111AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year Examinations, September - 2023

MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, CHEM, IT, MCT, AE, AME, MIE, PTM, AGE)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A**(25 Marks)**

- 1.a) Using Cayley - Hamilton theorem find inverse for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. [2]
- b) Define Index and Signature of a canonical form. [3]
- c) State Lagrange's mean value theorem. [2]
- d) In polar coordinates, if $x = r \cos \theta$, $y = r \sin \theta$ then show that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$. [3]
- e) Write the relation between beta and gamma functions. [2]
- f) Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of gamma function [3]
- g) Write the laws of natural growth and decay in mathematical form. [2]
- h) Find the particular integral of $(D^2 + 1)y = x^2$. [3]
- i) State any two properties of Laplace transform. [2]
- j) Find the Laplace transform of $(t-1)^2 u(t-1)$. [3]

PART - B**(50 Marks)**

- 2.a) Prove that $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is a unitary and find A^{-1} .

- b) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 2 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ by reducing it to the normal form. [5+5]

OR

- 3.a) Test for consistency and solve $5x+3y+7z=4, 3x+26y+2z=9, 7x+2y+10z=5$.
 b) Reduce the quadratic form $2x_1x_2+2x_1x_3-2x_2x_3$ into a sum of squares by an orthogonal transformation and also discuss its nature. [5+5]

- 4.a) Verify Rolle's theorem for $f(x)=x^2-6x+8$ in $[2,4]$.
 b) If $u=3x+2y-z, v=x-2y+z$ and $w=x(x+2y-z)$, show that they are functionally related, and find the relation. [5+5]

OR

- 5.a) Find the maximum and minimum values of $x^3+3xy^2-15x^2-15y^2+72x$.
 b) A rectangular box open at top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. [5+5]

- 6.a) Prove that $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta = \pi/\sqrt{2}$.
 b) Express $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx$ as a beta function. [5+5]

OR

- 7.a) Find by the double integration, the area lying inside the circle $r = a \sin \theta$ and outside the Cardioid $r = a(1 - \cos \theta)$.
 b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2} = 1$. [5+5]

- 8.a) A body kept in air with temperature $25^\circ C$ cools from $140^\circ C$ to $80^\circ C$ in 20 minutes. Find when the body cools down to $30^\circ C$.
 b) Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$. [5+5]

OR

- 9.a) Solve $(D^2 + 1)y = \sin x \sin 2x$.
 b) If a voltage of $20 \cos 5t$ is applied to a series circuit consisting of 10 ohm resistor and 2 henry inductor, determine the current at any time t. [5+5]

- 10.a) Find the Laplace transform of $e^{-t} \int_0^t \frac{\sin t}{t} dt$.
 b) Find the $L\{f(t)\}$, where $f(t) = \begin{cases} \cos(t-\pi/3), & \text{if } t > \pi/3 \\ 0, & \text{if } t < \pi/3 \end{cases}$ [5+5]

OR

- 11.a) Apply convolution theorem to find the inverse Laplace transforms of $\frac{1}{s(s^2+4)}$.
 b) Solve $(D^2 + 2D + 5)y = e^t \sin t$, given that $y(0)=0, y'(0)=1$ using Laplace transforms. [5+5]